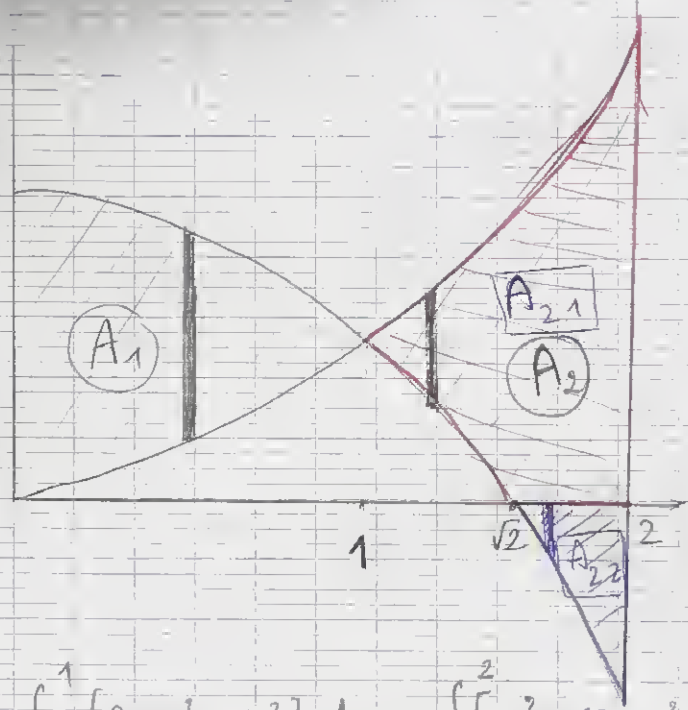
$y = x^2$ & $y = 2 - x^2$, $0 \leq x \leq 2$



why is the aerea in the negative partition is not considered ?

$$A = \int_0^1 [2 - x^2 - x^2]\, d_x + \int_1^2 [x^2 - (2 - x^2)]\, d_x = \checkmark$$

Proof

$$A_2 = A_{21} + A_{22}$$

$$A_{21} = \int_1^{\sqrt{2}} [x^2 - (2 - x^2)]\, d_x + \int_{\sqrt{2}}^2 x^2\, d_x$$

$$A_{22} = \int_{\sqrt{2}}^2 (0 - (2 - x^2))\, d_x = \int_{\sqrt{2}}^2 [-(2 - x^2)]\, dx$$

$$\therefore A_2 = \int_{\sqrt{2}}^2 [x^2 - (2 - x^2)]\, d_x + \int_1^{\sqrt{2}} [x^2 - (2 - x^2)]\, d_x$$

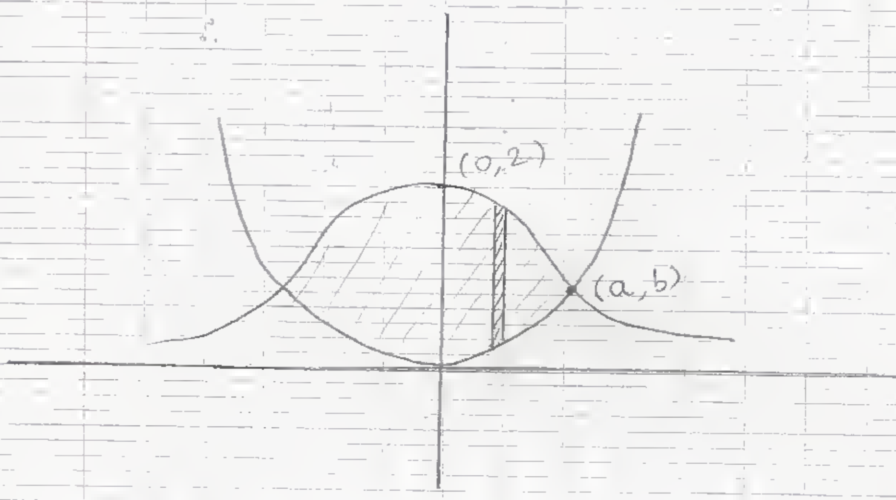
$$\cancel{A_2 = \int_1^2 (x^2 - (2 - x^2) + x^2 - 2 + x^2)\, d_x =}$$

$$\therefore A_2 = \int_1^2 [x^2 - (2 - x^2)]\, d_x \quad \checkmark$$

Area between $y = x^2$, $y = \frac{2}{x^2+1}$

I won't sketch a good graph just a rough

$y = \frac{2}{x^2+1} \rightarrow$ even f^n, max value is $(0,2)$

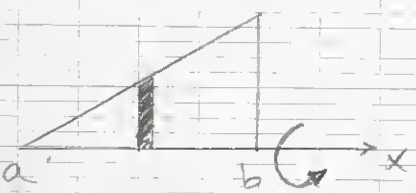


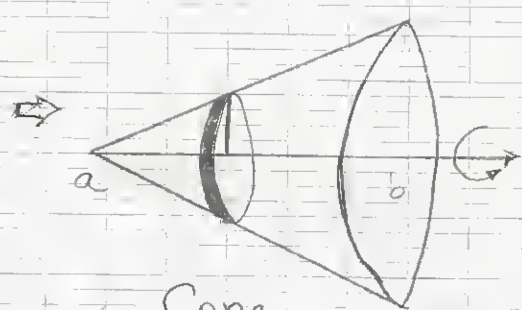
$$A = 2\int_0^a \left[\frac{2}{x^2+1} - x^2\right] dx = \pi - \frac{2}{3}$$

6.2 Volumes

"Volumes of solids got by revolving certain bounded area about (x-axis) or (y-axis)"

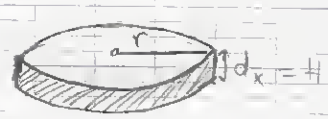
Ex:





Cone

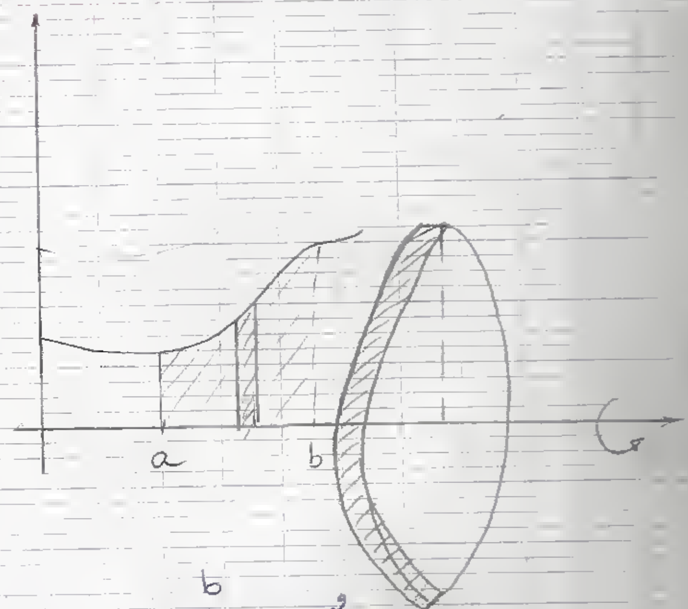
$$A = \int_a^b f(x)\,dx$$



$$A = \pi r^2 H$$

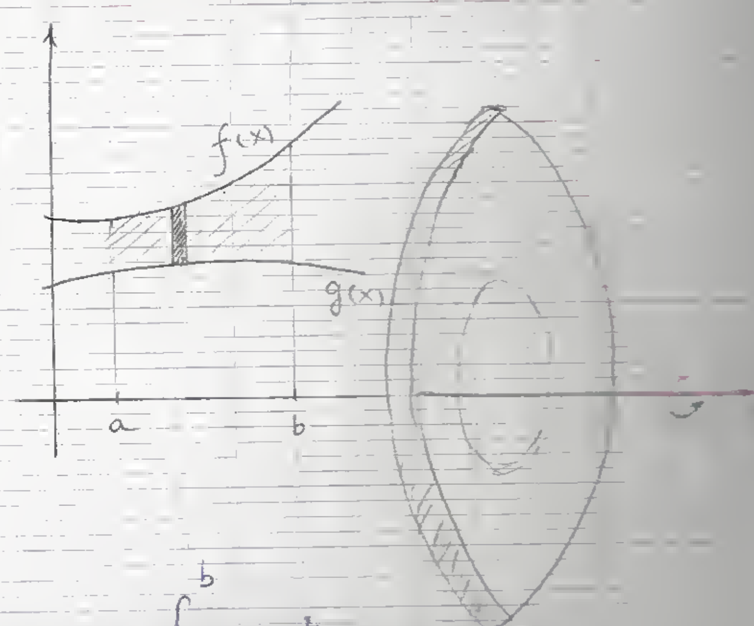
$$V = \int_a^b \pi (f(x))^2\,d_x = \pi \int_a^b (f(x))^2\,dx$$

1/ The Method of Disks:



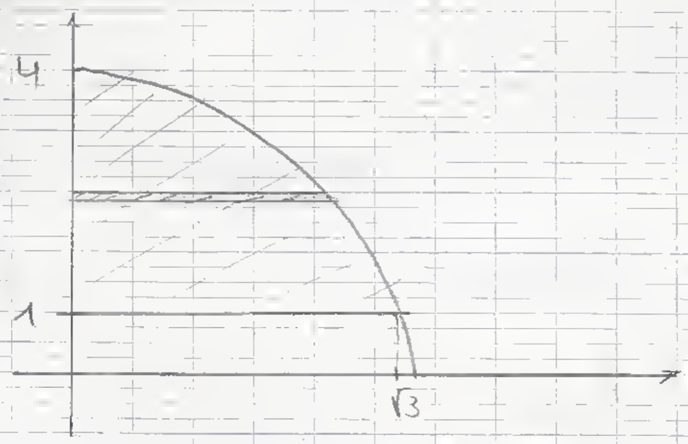
$$V = \pi \int_a^b [f(x)]^2 \, dx$$

2/ The Method of Washers:



$$V = \pi \int_a^b \left[[f(x)]^2 - (g(x))^2 \right] dx$$

$\underline{Ex}$: $y = 4 - x^2$; $y = 1$, $x = 0$, $x = \sqrt{3}$



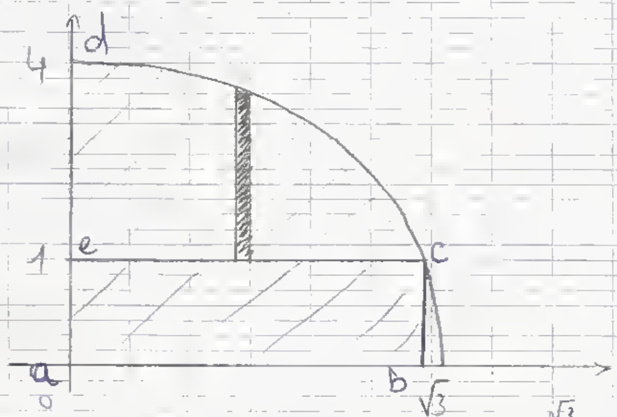
1, Volume about y-axis "Disk":

$$V = \pi \int_1^4 [\sqrt{4-y}\,]^2 \, dy = \checkmark$$

$x^2 = 4 - y$

$x = \pm\sqrt{4-y}$

2, Volume about x-axis "Washer":



$$V_1 = V_{abcd} = \pi \int_0^{\sqrt{3}} [4 - x^2]^2 \, d_x$$

$$V_2 = V_{abce} = \pi \, (1)^2 (\sqrt{3})$$

$$V = V_1 - V_2 = \pi \left[\int_0^{\sqrt{3}} [4 - x^2]^2 \, d_x \right] - \sqrt{3} \;]$$

$$= \pi \int_0^{\sqrt{3}} \left[(4 - x^2)^2 - (1)^2 \right] d_x$$